FACULTY OF MECHANICAL ENGINEERING CTU IN PRAGUE

DEPARTMENT OF ENVIRONMENTAL ENGINEERING

Ventilation / „Fun Vent“

10. Duct design

Bachelor / Master degree course

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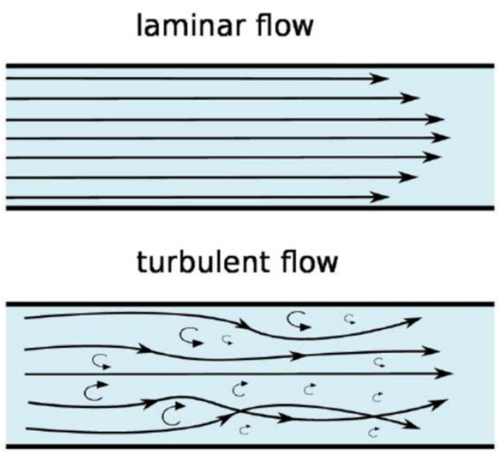
4/2022

Laminar and turbulent flows



Laminar and turbulent flows

Reynolds number (in circular duct)

$$\mathrm{Re} = \frac{wd}{\nu}$$

- laminar flow $Re \leq 2300$
- transitional flow $2300 < Re < 10000$
- fully turbulent flow $Re > 10\,000$

ν_{air}… kinematic viscosity $[m^2/s] = 14.5 \times 10^{-6}\ [m^2/s]$

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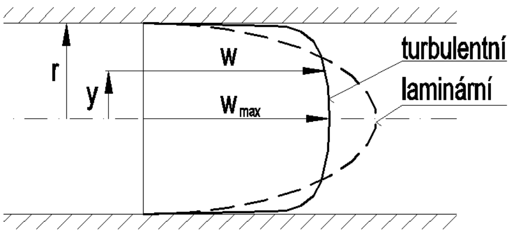
Laminar and turbulent flows

Flow characteristics



$$\frac{w}{w_{\max}} = \left(1 - \frac{y}{r}\right)^{1/n}$$

n …exponent f(Re)

$$V = w_s S$$

$$w_s = \frac{1}{\pi r^2} \int_S w dS$$

$$w_s = \frac{1}{\pi r^2} \int_S w_{\max} \left(1 - \frac{y}{r}\right)^{1/n} 2\pi y dy$$

$$\frac{w_s}{w_{\max}} = 0{,}817$$

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Pressure losses

Bernoulli equation

$$p_{s1} + h_1 \rho g + \frac{\rho}{2} w_1^2 = p_{s2} + h_2 \rho g + \frac{\rho}{2} w_2^2 + \Delta p$$

Pressures in the duct

$$p = p_s + p_d = p_s + \frac{w^2}{2}\rho$$

$$\Delta p = \left(p_{s1} + \frac{\rho}{2} w_1^2\right) - \left(p_{s2} + \frac{\rho}{2} w_2^2\right) = p_{t1} - p_{t2}$$

Pressure losses

- friction
- local pressure losses

$$\Delta p = \underbrace{\lambda \frac{l}{d}\frac{w^2}{2}\rho}_{\text{friction}} + \underbrace{\sum \zeta \frac{w^2}{2}\rho}_{\text{local losses}} = R.l + Z \qquad w = \frac{\dot{V}}{A} = \frac{4\dot{V}}{\pi d^2}$$

$$\Delta p = \left(\lambda \frac{l}{d} + \sum \zeta\right) \frac{w^2}{2}\rho = k\dot{V}^2$$

$$k = \left(\frac{\lambda \cdot l}{d} + \sum \zeta\right) \frac{8 \cdot \rho}{\pi^2 \cdot d^4}$$

Friction losses

Laminar flow

$$\lambda = \frac{64}{Re}$$

Turbulent flow

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{\varepsilon / d}{3.71} + \frac{2,51}{Re\sqrt{\lambda}}\right)$$

Colebrook (1939)

ε/d – relative roughness

$$\lambda = \frac{0.0812}{Re^{0.125} d^{0.11}}$$

Smolik (1959) for $\varepsilon = 0.15$

Note:
Darcy-Weisbach λ
Fanning friction factor f

$$\lambda = 4f$$

Friction losses

Turbulent flow

$$\lambda = \frac{0.3164}{\sqrt[4]{\mathrm{Re}}}$$

for smooth pipes and duct (plastic)

5000 < Re < 80 000

Friction losses

Roghness height of the conduit wall surfaces

Material	ε (mm)
Galvanized steel	0.15
Concrete duct – smooth surface	0.5
Concrete duct – rough surface	1.0 – 3.0
Smooth brass, copper	0.015
Hose pipe	0.6 - 3
Plastic pipe	0.007

Friction losses

Hydraulic diameter

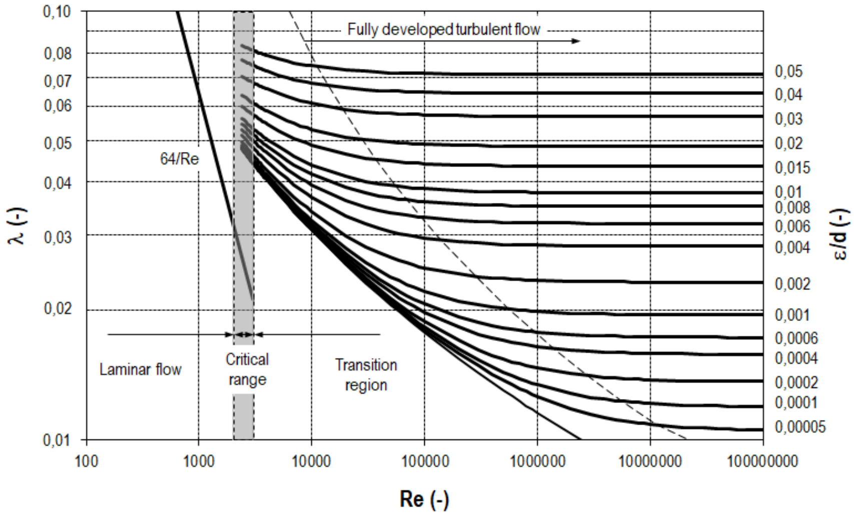
$$d_h = \frac{4A}{O} = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

Rectangular ducts - correction

$$\lambda = C\lambda_d$$

$$C = 1.1 - 0.1\frac{b}{a}$$

Moody chart



Local pressure losses

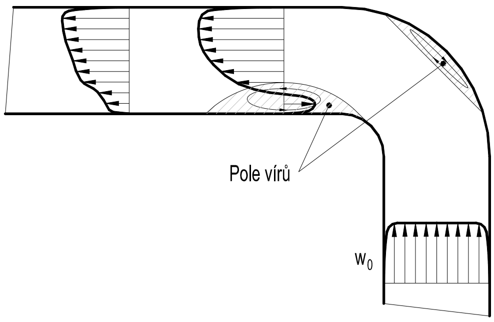
Local pressure losses are caused by the fluid flow through the duct fittings:

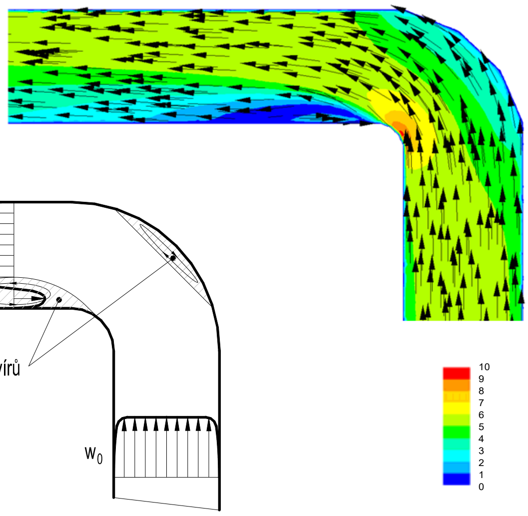
- which change the direction of the flow (elbows, bands, etc.)
- affect the flow in the straight duct with constant cross-section (valves, stopcocks, filters etc.).

$$\Delta p_l = \sum \zeta p_d = \sum \zeta \frac{w^2}{2} \rho$$

- ζ ... local loss coefficient (experiments - see Idelchik 1986)
- experimental data (except Borda loss)

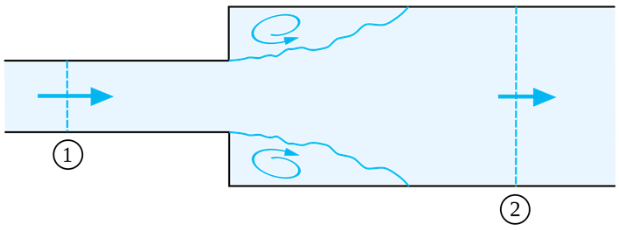
Local pressure losses





Borda loss

Flow expansion

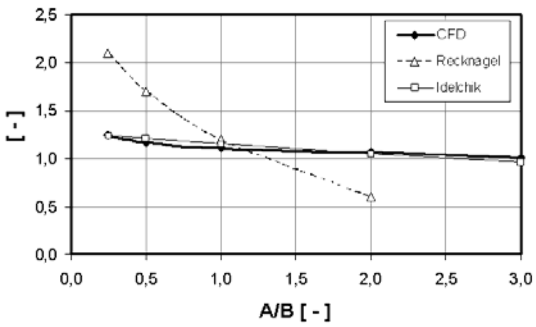


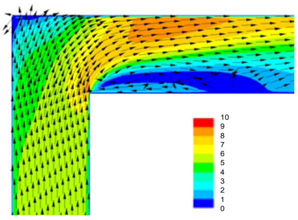
Borda – Carnot eq.

$$\Delta p_l = \left(\frac{A_2}{A_1} - 1\right)^2 \frac{w_2^2}{2}\rho$$

Local pressure losses - example

$$\zeta = 1.11\left(\frac{a}{b}\right)^{-0.08}$$





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Duct design

Methods

- velocity method
- equal-friction method
- static regain method

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Velocity method

Duct design procedure:

1) Find the main line

Rule no. 1: the main line is the maximum pressure loss line

(longest line, most segment line (?))

2) Air flow rate *V* (m^3/h) in duct sections is known

3) Selection of the air velocity in the duct *w*

Rule no. 2: Air velocity increase towards the fan

Velocity method

	Air velocity *w* (m/s)			
	Main section		Side section	
Ventilation and low-pressure air-conditioning	recomend.	max.	recomend.	max.
- residential buildings	3.5 - 5	6	3	5
- public buildings	5 – 7	8	3 – 4.5	6.5
- industry	6 - 9	11	4 - 5	9
High-pressure air-conditioning	8 - 12	15 - 20	8 - 10	18

Velocity method

4) duct area A (m^2) $\rightarrow$ diameter d or $a \times b$

$$d = \sqrt{\frac{4\dot{V}}{\pi w}}$$

$\rightarrow$ nominal diameter d_N or $a_N \times b_N$

Rule no. 3: Duct sizes: 80, 100, 125, 140, 160, 180, 200, 250, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800, 2000

Velocity method

5) $d_N \rightarrow$ real velocity w_{real}

$$w_{real} = \frac{4V}{\pi d_N^2}$$

6) calculation of dynamic pressure p_d

7) Reynolds number $\rightarrow$ friction coefficient λ

8) local loss coefficients ζ

9) pressure loss of the duct section $\Delta p_{z,i}$

$$\Delta p_i = \left(\lambda \frac{l_i}{d_i} + \sum \zeta \right) \frac{w_i^2}{2} \rho$$

Velocity method

Rule no. 4: Balancing

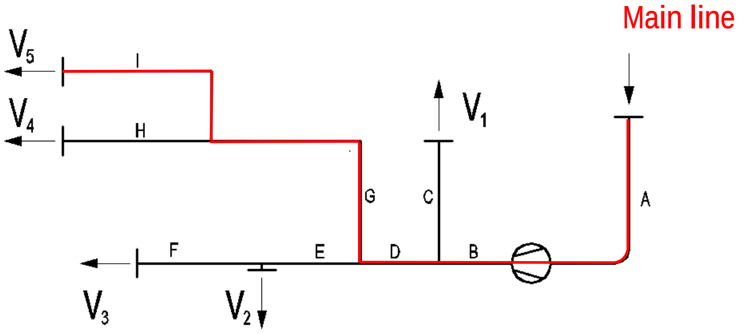
$$\Delta p_F + \Delta p_E = \Delta p_G + \Delta p_I$$

10) total pressure loss is the sum of the duct sections pressure losses

$$\Delta p = \sum \Delta p_i$$

Velocity method

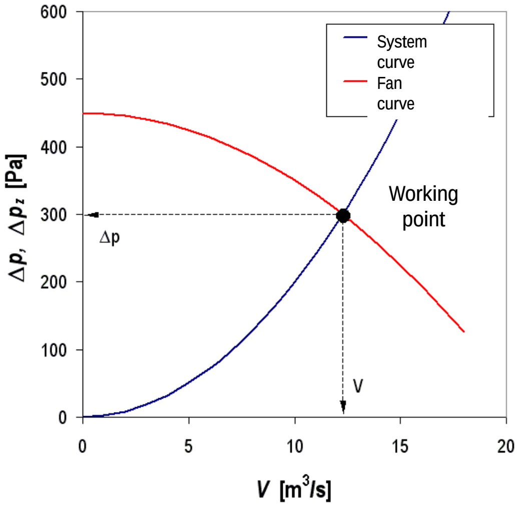
Main line



$$\Delta p = \Delta p_A + \Delta p_B + \Delta p_D + \Delta p_G + \Delta p_I$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5$$

System and fan curve



Example

Example 1: Dimension the air duct system. Use the velocity method.

air velocity w = 6 - 10 m/s,

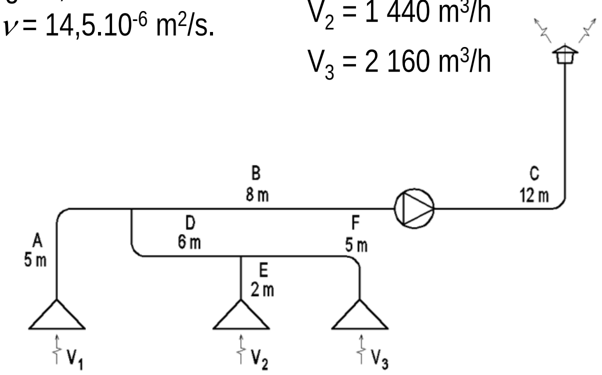
air density ρ = 1,2 kg/m³,

kinematic viscosity ν = 14,5.10^{-6} m²/s.

V_1 = 9 000 m³/h

V_2 = 1 440 m³/h

V_3 = 2 160 m³/h



Example

Example 1:

$$D_{calc} = \sqrt{\frac{4V}{\pi w}} \quad ⇨ \quad D_N \quad ⇨ \quad w_{real} = \frac{4V}{\pi D_N^2}$$

$$⇨ \quad \mathrm{Re} = \frac{w_{realt} D_N}{\nu} \quad ⇨ \quad \lambda = \frac{0{,}0812}{\mathrm{Re}^{0{,}125} D_N^{\;0{,}11}}$$

$$\Delta p_f = \lambda \frac{l}{D} \frac{w_{real}^{\;2}}{2} \rho \qquad \Delta p_l = \sum \zeta \frac{w_{real}^{\;2}}{2} \rho$$

$$\Delta p_{t,i} = \Delta p_f + \Delta p_l (+\Delta p_{el})$$

Example

Line	l	V	V	w_{calc}	D_{calc}	D_N	w_{real}	p_d	Re	l	Δp_f	$\Sigma\zeta$	Δp_l	Δp_{el}	Δp_t
-	m	m³/h	m³/s	m/s	mm	mm	m/s	Pa	-	-	Pa	-	Pa	Pa	Pa
												0,41		19	
												0,96		0	
												0,46		0	
												2,04		0	
TOTAL															XX

Equal-Friction Method

Duct design procedure:

1) selection of pressure loss per unit length R = 0,8 – 4 Pa/m

$$R = \lambda \frac{1}{d} \frac{w^2}{2} \rho$$

2) local pressure losses $\rightarrow$ friction in straight duct with equivalent length

$$\lambda \frac{l_e}{d} \frac{w^2}{2} \rho = \zeta \frac{w^2}{2} \rho \Rightarrow l_e = \frac{\zeta}{\lambda} d$$

3) duct section pressure loss

$$\Delta p = R\left(l + l_e\right)$$

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Equal-Friction Method

Friction chart

Choice:

R = 1 Pa/m

Air flow rate:

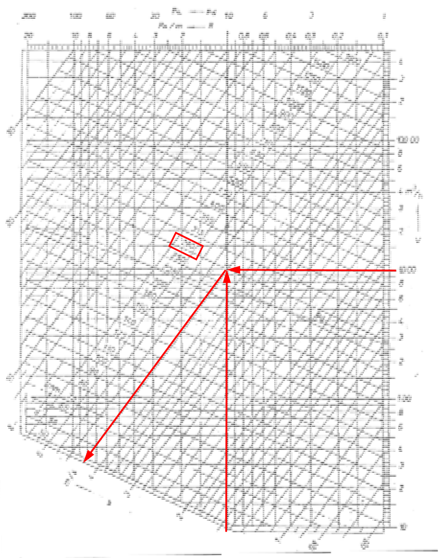
1 000 m^3/h

diameter D:

280 mm

Velocity :

w = 4,5 m/s



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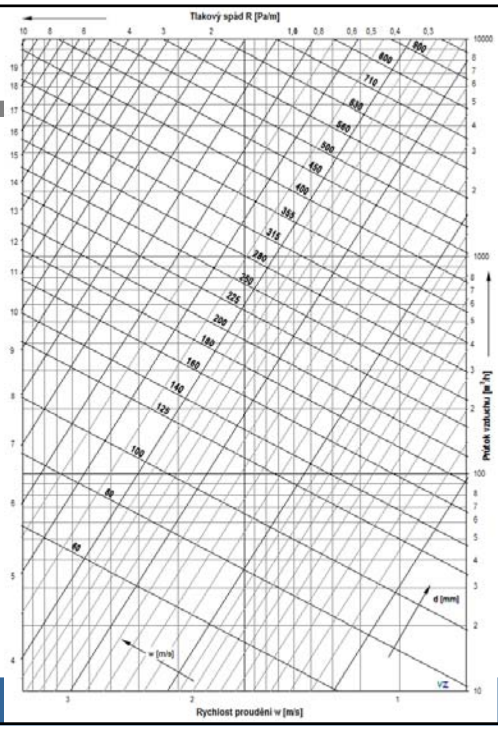


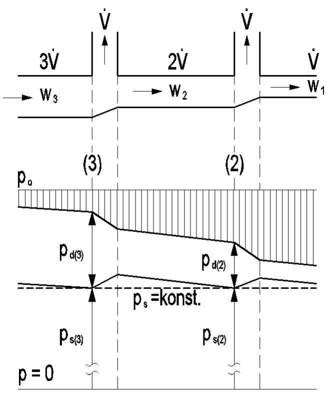
Static Regain Method

- for uniform air supply
- constant static pressure before the branch

Principle

- cross section reduction after branches to change the dynamic pressure
- decreasing of dynamic pressure balances the pressure losses in the duct section

$$\Delta p_{z3-2} = p_{d3} - p_{d2}$$



Static Regain Method

Assumptions:

- V_i = const.
- b = const.
- $i = n, n - 1, \dots 1$
- calculation of dimension a

$$a_{i-1} = a_i \sqrt{1 + \frac{\lambda_{i-1}}{d_{i-1}} l_{i-1} \frac{i-1}{i}}$$

Static Regain Method

Deduction:

$$\Delta p_{z2-1} = p_{d2} - p_{d1}$$

$$\lambda \frac{l_1}{d_1} \frac{w_1^2}{2} \rho = \frac{w_2^2}{2} \rho - \frac{w_1^2}{2} \rho \quad \text{kde} \quad w_1 = \frac{\dot{V}_1}{a_1 b}, w_2 = \frac{\dot{V}_2}{a_2 b}$$

$$\lambda \frac{l_1}{d_1} \frac{\dot{V}_1^2}{a_1^2 b^2} = \frac{\dot{V}_2^2}{a_2^2 b^2} - \frac{\dot{V}_1^2}{a_1^2 b^2}$$

Static Regain Method

$\Rightarrow$

$$\lambda \frac{l_1}{d_1}\frac{\dot V_1^2}{a_1^2} = \frac{\dot V_2^2}{a_2^2} - \frac{\dot V_1^2}{a_1^2} \qquad \leftarrow \qquad \dot V_1 = \dot V, \dot V_2 = 2\dot V$$

$$\lambda \frac{l_1}{d_1}\frac{1^2 \cdot \dot V^2}{a_1^2} + \frac{1^2 \cdot \dot V^2}{a_1^2} = \frac{2^2 \cdot \dot V^2}{a_2^2} \quad \Rightarrow \quad \lambda \frac{l_{i-1}}{d_{i-1}}\frac{(i-1)^2}{a_{i-1}^2} + \frac{(i-1)^2}{a_{i-1}^2} = \frac{i^2}{a_i^2}$$

$$a_i^2\left(1+\lambda\frac{l_{i-1}}{d_{i-1}}\right)(i-1)^2 = a_{i-1}^2 i^2 \quad \Rightarrow \quad a_{i-1} = a_i\sqrt{1+\frac{\lambda_{i-1}}{d_{i-1}}l_{i-1}}\,\frac{i-1}{i}$$

Duct systems

Shapes
- rectangular
- round
- flexible duct

Materials
- steel galvanized
- aluminium
- plastic PVC
- textile
- ALP

Duct systems

Duct leakage rate

$$\dot{V} = m \Delta p^{0,67} A_d$$

where A_d ... duct surface [m²]

Class	Charakteristics of the leakage path m [m³/s per m²]
A	$0.027 \cdot 10^{-3}$
B	$0.009 \cdot 10^{-3}$
C	$0.003 \cdot 10^{-3}$
D	$0.001 \cdot 10^{-3}$

Thermal insulation

Purpose

- condensation risk
- heat losses/gains

Thickness of TI

- indoor 45 – 60 mm
- outdoor 80 – 100 mm (with sheet covering)

FAKULTA STROJNÍ ČVUT V PRAZE

ÚSTAV TECHNIKY PROSTŘEDÍ

Thank you for your attention

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