FACULTY OF MECHANICAL ENGINEERING CTU IN PRAGUE

DEPARTMENT OF ENVIRONMENTAL ENGINEERING

Environmental Engineering

2. Psychrometrics

Bachelor degree course

Vladimír Zmrhal

Winter semester 10/2023

Dry air

Components in Dry Air	Volume Ratio compared to Dry Air	Molar Mass *M* [kg/kmol]	Molar Mass in Air
Oxygen	0.2095	32.00	6.704
Nitrogen	0.7809	28.02	21.88
Carbon Dioxide	0.0003	44.01	0.013
Hydrogen	0.0000005	2.02	0
Argon	0.00933	39.94	0.373
Neon	0.000018	20.18	0
Helium	0.000005	4.00	0
Krypton	0.000001	83.8	0
Xenon	$0.09\ 10^{-6}$	131.29	0
Total Molar Mass of Air			28.97

Water vapor

- is almost always presence in the air
- the amount of water vapor varies from zero (dry air) to a maximum that depends on temperature and pressure
- water vapor in air will replace other gases and reduce the total density of the mixture
- dry air is more dense than moist air !

Moist air

- the mixture (two-component) of dry air and water vapour
- compared to the other gases in the air water may condense

Saturation

- is a state of neutral equilibrium between moist air and the condensed water phase (liquid or solid)
- the maximum vapor pressure possible before the vapor start to condense at an actual temperature - is called the saturation pressure – $p_{v,s}$

Dalton's law – Eq. of state

Moist air

- Dalton's law (1801) of parcial pressures

$$p = \sum_i p_i = p_a + p_v$$

where

p_a … partial pressure of dry air [Pa]

p_v … partial pressure of water vapor [Pa]

- equation of state (ideal gas law)

$$pV = MrT$$
$$pv = rT$$

$$v = \frac{1}{\rho} = \frac{V}{M}$$ …specific volume

Dalton's law – Eq. of state

$$p_a V = M_a r_a T$$
$$p_v V = M_v r_v T$$

$$r_a = \frac{R}{m_{ma}} = \frac{8314}{28.96} = 287.1 \quad \text{[J/kg.K]}$$

$$r_v = \frac{R}{m_{mv}} = \frac{8314}{18.02} = 461.4 \quad \text{[J/kg.K]}$$

where

m … molar mass [kg/kmol]

Thermodynamic properties

Dry air and water vapor

• universal gas constant	R = 8314 [J/kmol.K]
• gas constant for dry air	r_a = 287.1 [J/kg.K]
• gas constant for water vapor	r_v = 461.5 [J/kg.K]
• latent heat vaporization of water (for 0 °C)	l = 2500 [kJ/kg]
• specific heat capacity of dry air	c_a = 1010 [J/kg.K]
• specific heat capacity of water vapor	c_v = 1860 [J/kg.K]
• density of air	= 1.2 kg/m3

Subscripts: a = dry air, v = water vapor, w = water
s = saturated

7

Humidity of moist air

➢ relative humidity

$$\varphi = \frac{p_v}{p_{v,s}} \qquad [\%]$$

➢ humidity ratio

$$x = \frac{M_v}{M_a} = \frac{r_a p_v}{r_v p_a} = 0.622 \frac{p_v}{p_a} = 0.622 \frac{\varphi p_{v,s}}{p - \varphi p_{v,s}} \qquad [kg_v/kg_{da}]$$

➢ absolute humidity (water vapor concentration)

$$a = \rho_v = \frac{M_v}{V} = \frac{p_v}{r_v T} \qquad [kg/m^3]$$

8

Density of moist air

from Dalton´s law …

$$\rho = \rho_a + \rho_v = \frac{M_a}{V} + \frac{M_v}{V} = \frac{p_a}{r_a T} + \frac{p_v}{r_v T} = \frac{p - p_v}{r_a T} + \frac{p_v}{r_v T}$$

$$= \frac{1}{r_a T}\left(p - p_v\left(1 - \frac{r_a}{r_v}\right)\right)$$

$$\rho = \frac{1}{r_a T}\left(p - 0.378\varphi p_{v,s}\right)$$

➢ the moist air is lighter than dry air !!!

9

Enthalpy of moist air

➢ the sum of the individual partial enthalpies of the components

➢ 1 kg of dry air + x kg/kg$_{da}$ of saturated water vapor

$$h = h_a + x h_v \qquad [\text{kJ/kg}_{da}]$$

$$h = c_a t + x\left(l + c_v t\right) = 1.01t + x\left(2500 + 1.86t\right)$$

➢ enthalpy relate to 0 °C

$h(t = 0\ °C;\ x = 0\ \text{kg}_v/\text{kg}_{da}) = 0$ [kJ/kg$_{da}$]

10

Water vapor saturation pressure

- for t = -20 to 0°C

$$\ln p_{v,s} = 28.926 - \frac{6148}{273.1 + t}$$

- for t = 0 to 80 °C

$$\ln p_{v,s} = 23.58 - \frac{4044.2}{235.6 + t}$$

Water vapor saturation pressure

- according to ASHRAE Handbook

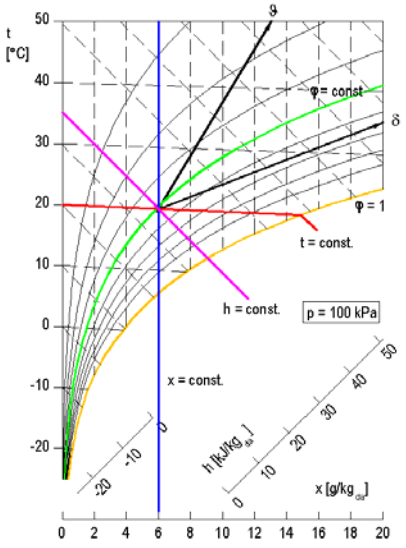
$$\ln p_{v,s} = \frac{C_1}{T} + C_2 + C_3 \cdot T + C_4 \cdot T^2 + C_5 \cdot T^3 + C_6 \cdot T^4 + C_7 \cdot \ln(T)$$

for t = -100 to 0 °C	for t = 0 to 200 °C
$C_1 = -5.6745359.10^3$	$C_1 = -5.8002206.10^3$
$C_2 = 6.3925247$	$C_2 = 1.3914993$
$C_3 = -9.6778430.10^{-3}$	$C_3 = -4.8640239.10^{-2}$
$C_4 = 6.2215701.10^{-7}$	$C_4 = 4.1764768.10^{-5}$
$C_5 = 2.0747825.10^{-9}$	$C_5 = -1.4452093.10^{-8}$
$C_6 = -9.484024.10^{-13}$	$C_6 = 0$
$C_7 = 4.1635019$	$C_7 = 6.5459673$

Molliere h-x diagram

- graphic representation of the relationship between air temperature, moisture content and enthalpy
- basic design tool for building engineers

Example 1a: Find out the air state
t = 34 °C
h = 60 kJ/kg
x = ?
φ = ?
p_v = ?



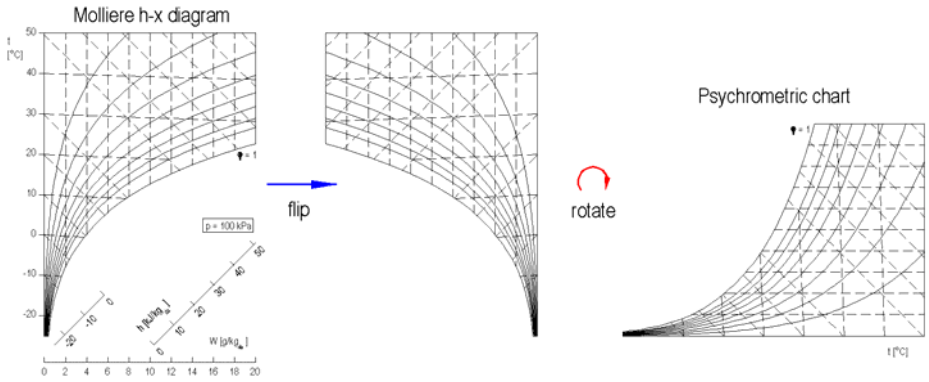
Molliere h-x diagram

Example 1b:
Calculate the air state. Compare the results with h-x diagram.

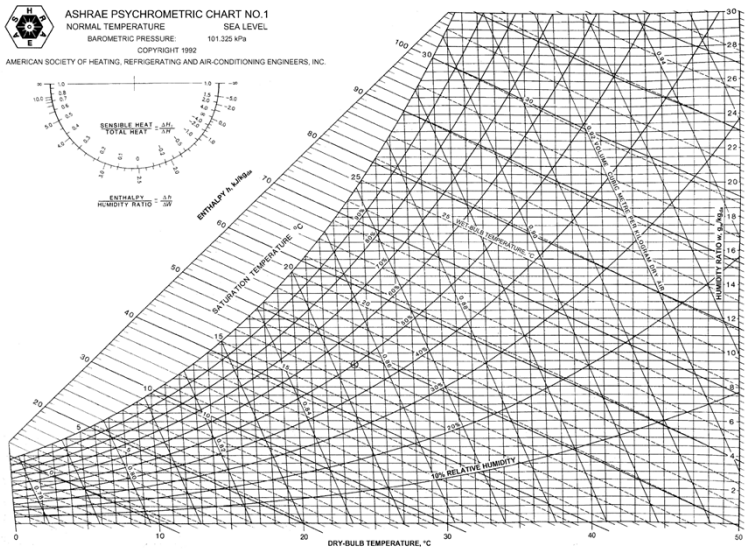
t = 20 °C
φ = 40 %
p = 100 kPa

$p_{v,S}$ = ?
p_v = ?
x = ?
h = ?
ρ = ?

Molliere vs. Psychrometric chart



ASHRAE Psychrometric chart



Molliere h-x diagram

Direction of the air-conditioning process δ

- is used to establish the direction of a condition on the psychrometric chart

$$\delta = \frac{\Delta h}{\Delta x} = \frac{h_2 - h_1}{x_2 - x_1}$$

Room sensible heat factor (RSHF) ϑ

- expresses the ratio between sensible heat load and total heat load in the room

$$\vartheta = \frac{\dot{Q}_{sen}}{\dot{Q}_{total}} = \frac{\dot{Q}_{sen}}{\dot{Q}_{sen} + \dot{Q}_{lat}} = \frac{c\Delta t}{\Delta h}$$

Molliere h-x diagram

Dry bulb temperature t

- usually referred to as air temperature, is the air property that is most common used $t_{db} = t$

Dew-point temperature t_{dp}

- is the temperature at which water vapor starts to condense out of the air (the temperature at which air becomes completely saturated)

Wet bulb temperature t_{wb}

- this is the temperature indicated by a moistened thermometer bulb exposed to the air flow
- the temperature of adiabatic saturation

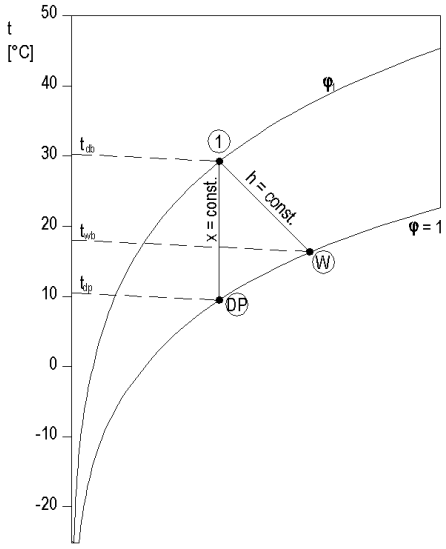
Molliere h-x diagram

Psychrometric equation

$$p_v = p_{v,s,wb} - Ap(t - t_{wb})$$

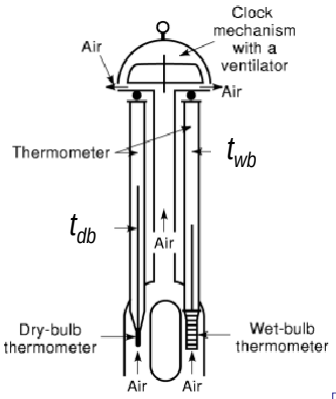
$$A = 662 \cdot 10^{-6} \qquad [K^{-1}]$$

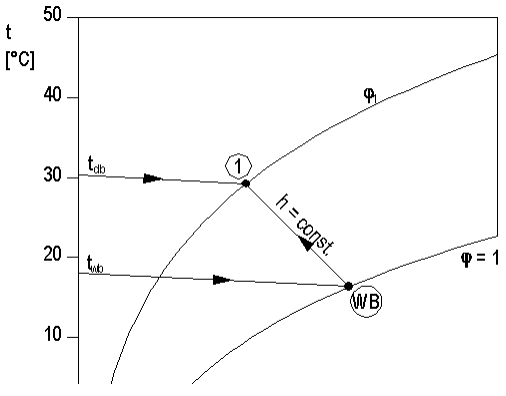
Example 2:
t = 30 °C
h = 56 kJ/kg
t_{wb} = ?
t_{dp} = ?



Molliere h-x diagram

Assman psychrometer





Example 3: t_{db} = 34 °C, t_{wb} = 18 °C, φ = ?

Dimensioning of air-conditioning

Outdoor air conditions (Czech Republic)

WINTER

$t_e = t_{e,loss}$ – 3 °C, φ_e = 100 % ($t_{e,loss}$ = -12, -15, -18 °C)

SUMMER

t_e = 32 °C, h_e = 58 kJ/kg

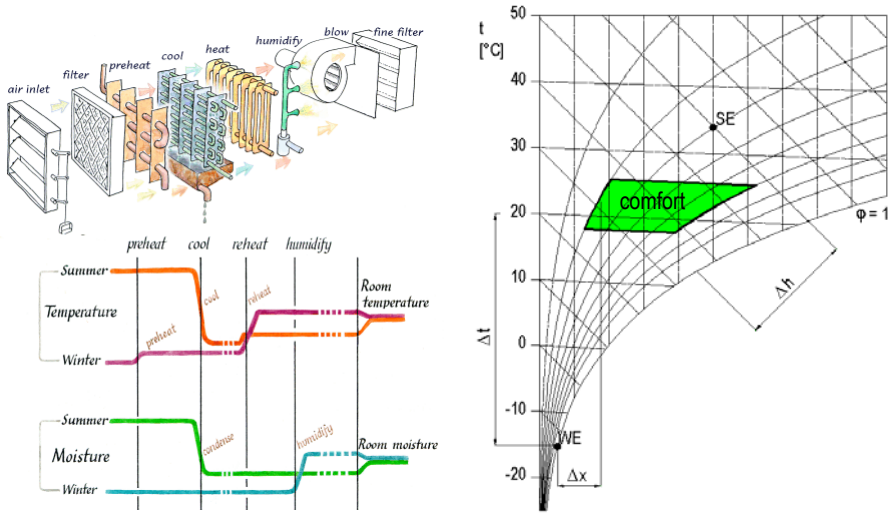
Indoor air conditions (for air-conditioned spaces)

Indoor air temperature t_i = 18 to 26 °C

Relative humidity φ_i = 30 to 70 %

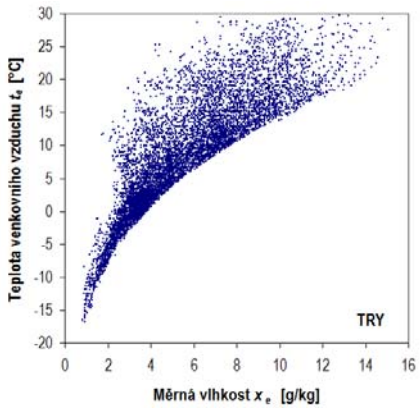
Example 4: Find the summer and winter extreme conditons in the diagram

Air-conditioning processes



Air-conditioning processes

Typical climatic data (Czech Republic)



TRY – test reference year

t_e – ambient air temperature [°C]

x_e – humidity ratio [g/kg]

Mixing of two moist airstreams

Moisture balance

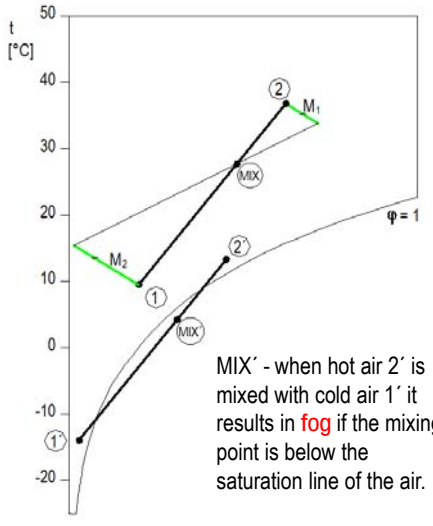
$$\dot{M}_1 x_1 + \dot{M}_2 x_2 = (\dot{M}_1 + \dot{M}_2) x_m$$

$$x_m = \frac{\dot{M}_1 x_1 + \dot{M}_2 x_2}{\left(\dot{M}_1 + \dot{M}_2\right)}$$

Enthalpy balance

$$\dot{M}_1 h_1 + \dot{M}_2 h_2 = (\dot{M}_1 + \dot{M}_2) h_m$$

$$h_m = \frac{\dot{M}_1 h_1 + \dot{M}_2 h_2}{\left(\dot{M}_1 + \dot{M}_2\right)}$$



MIX´ - when hot air 2´ is mixed with cold air 1´ it results in fog if the mixing point is below the saturation line of the air.

Mixing of two moist airstreams

$\Rightarrow$ physical evidence

$$\dot{M}_1(x_1 - x_m) = \dot{M}_2(x_m - x_2)$$

$$\dot{M}_1(h_1 - h_m) = \dot{M}_2(h_m - h_2)$$

$$\delta = \frac{\Delta h}{\Delta x} = \frac{h_m - h_2}{x_m - x_2} = \frac{h_1 - h_m}{x_1 - x_m}$$

$$\frac{\dot{M}_1}{\dot{M}_2} = \frac{(x_m - x_2)}{(x_1 - x_m)} \qquad \frac{\dot{M}_1}{\dot{M}_2} = \frac{(h_m - h_2)}{(h_1 - h_m)}$$

Mass flow rate
$M = V.\rho$ [kg/s]

Example 5:
M_1 = 3 kg/s
t_1 = -15 °C
φ_1 = 100 %

M_2 = 2 kg/s
t_2 = 20 °C
x_2 = 6 g/kg

t_m = ?
x_m = ?

25

Air heating

Heating coil

- water coil heat exchangers

electric heaters

direct heaters

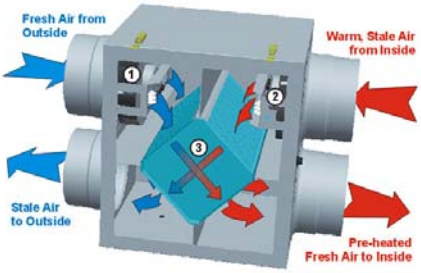
26

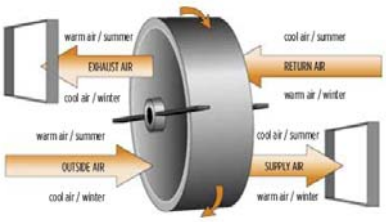
Air heating

Heat (energy) recovery

heat recuperation

heat regeneration





Air heating

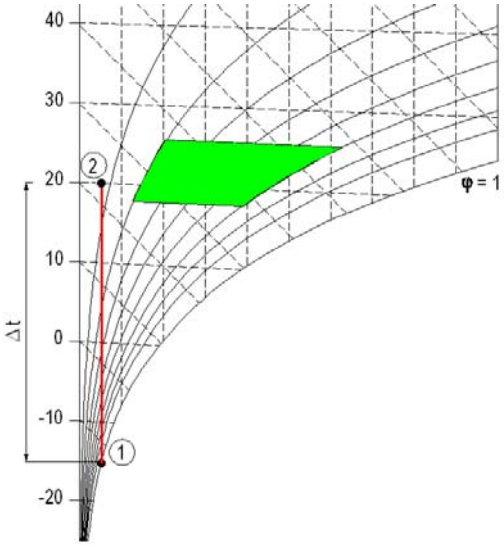
Heating coil capacity

$$\dot{Q}_h = \dot{V}\rho c\left(t_2 - t_1\right)$$

$$\Delta x = 0$$

$$\delta = \infty$$

Example 6:
V = 10 000 m³/h
t_1 = -15 °C
t_2 = 20 °C
Q_h = ?



Air cooling

Sensible cooling - without condensation („dry cooling“)

➢ the temperature on a cooling surface is above or equal to the dew point temperature → $t_c > t_{dp}$

Latent cooling - dehumidifying moist air

➢ the temperature on a cold surface is lower than the dew point temperature → $t_c < t_{dp}$

Cold surface temperature

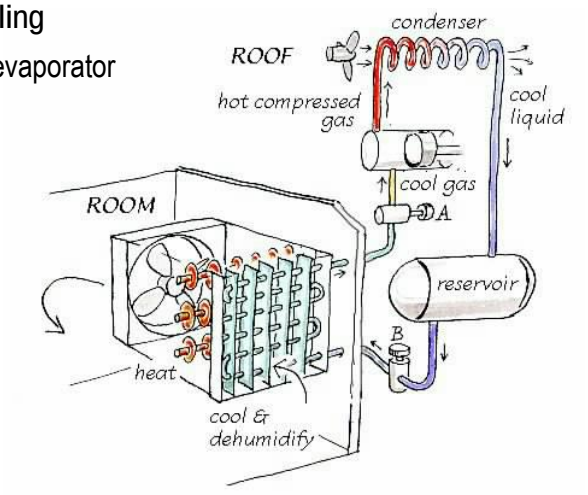
➢ water exchangers e.g. 6/12 °C $t_c = (t_{w1} + t_{w2})/2 = 9$ °C

➢ evaporators $t_c \approx 4$ to 5 °C

Air cooling

Direct air cooling

cooling coil = evaporator



Air cooling

Cooling coil capacity

- total heat flow rate through the cooling coil

$$\dot{Q}_c = \dot{Q}_{sen} + \dot{Q}_{lat} = \dot{V}\rho c\left(t_1 - t_2\right) + \dot{V}\rho l\left(x_1 - x_2\right)$$

$$\dot{Q}_c = \dot{V}\rho\left(h_1 - h_2\right)$$

$$\Delta h = c\Delta t + l\Delta x$$

Air cooling

Cooling without condensation

- $t_c > t_{dp}$

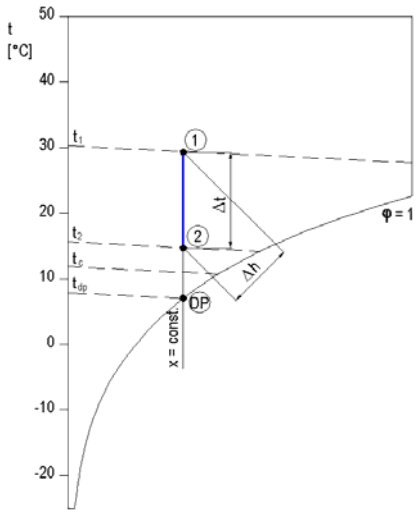
- cooling coil capacity

$$\dot{Q}_{sen} = \dot{V}\rho c\left(t_1 - t_2\right)$$

$$\dot{Q}_{lat} = 0$$

$$\dot{Q}_c = \dot{V}\rho\left(h_1 - h_2\right) =$$

$$= \dot{V}\rho c\left(t_1 - t_2\right)$$



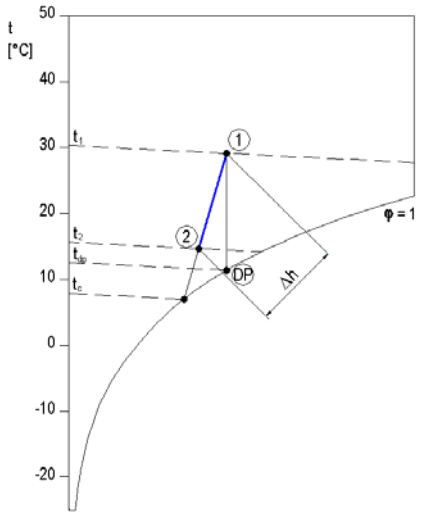
Air cooling

Cooling with dehumidifying of moist air

- $t_c < t_{dp}$
- cooling coil capacity

$$\dot{Q}_c = \dot{V}\rho(h_1 - h_2)$$

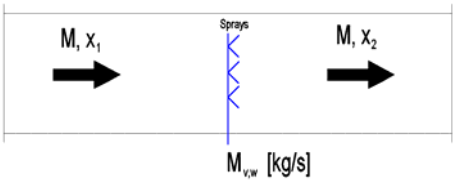
- sensible heat
- latent heat

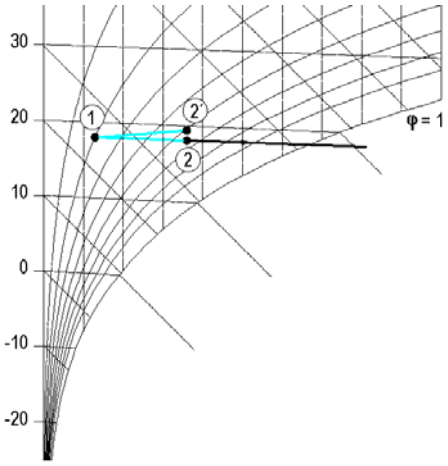


Air cooling

Cooling without condensation	Cooling with dehumidifying
Example 7a: V = 10 000 m³/h t_1 = 32 °C h_1 = 58 kJ/kg t_2 = 20 °C t_c = 16 °C Q_c = ?	Example 7b: V = 10 000 m³/h t_1 = 32 °C h_1 = 58 kJ/kg t_2 = 20 °C t_{w1}/t_{w2} = 6/12 °C Q_c = ?

Humidifying (adding steam)





Moisture balance

$$\dot{M}x_1 + \dot{M}_v = \dot{M}x_2$$

Heat balance

$$\dot{M}\ h_1 + \dot{M}_v h_v = \dot{M}\ h_2$$

Humidifying (adding steam)

Enthalpy of water vapor

$$h_v = l + c_v t = 2500 + 1.86 \cdot t \cong 2540 \qquad \text{[kJ/kg]}$$

Direction of the process

$$\delta = \frac{\Delta h}{\Delta x} = \frac{h_2 - h_1}{x_2 - x_1} = h_v$$

$$\delta = h_v = 2.5 \qquad \text{[kJ/g]}$$

Humidifying (adding steam)

Enthalpy of water vapor – reality

I = 2500 kJ/kg … for 0 °C

$$h_v = I + c_v t + c_w t = 2500 + 1.86 \cdot 100 + 4.2 \cdot 0 \cong 2680 \quad \text{[kJ/kg]}$$

I = 2257 kJ/kg … for 100 °C

$$h_v = I + c_v t + c_w t = 2257 + 2.08 \cdot 0 + 4.2 \cdot 100 \cong 2680 \quad \text{[kJ/kg]}$$

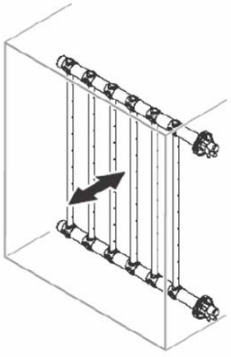
$$\delta = h_v = 2.68 \quad \text{[kJ/g]}$$

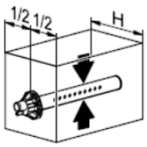
In reality the air is also heated !

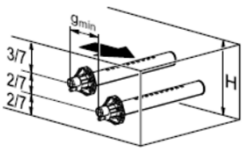
37

Humidifying (adding steam)

Distribution of steam (sprays)







Example 8:
V = 10 000 m^3/h
x_1 = 1 g/kg
x_2 = 5 g/kg
M_v = ?
Q_{hum} = ?

38

Humidifying (adding water - injection)

Adding water (100% evaporation)

$$\delta = \frac{\Delta h}{\Delta x} = \frac{h_2 - h_1}{x_2 - x_1} = h_w$$

Enthalpy of water

$$h_w = c_w t = 4.187t = 0 \text{ to } 420 \quad \text{[kJ/kg]}$$

Direction of the process

$$\delta = h_w \cong 0 \quad \text{[kJ/g]}$$

Humidifying (adding water)

Adiabatic cooling (air washer)

$t_{w1} = t_{w2}$

$Q = 0$

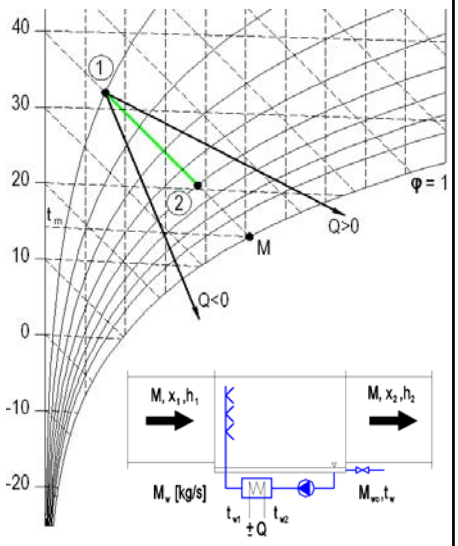
$\delta = c_w t_w = 4.187 t_w \cong 0$ kJ/g

Polytropic change

$t_{w1} \neq t_{w2}$

$Q > 0 \quad \ldots \delta > 0$

$Q < 0 \quad \ldots \delta < 0$



Humidifying (adding water)

Efficiency of air washer

$$\eta = \frac{x_2 - x_1}{x_{wb} - x_1}$$

Example 9:
η = 60 %
t_1 = 31 °C
φ_1 = 40 %
x_2 = ?
φ_2 = ?

Dehumidifying

Cooling of air - vapor condensation

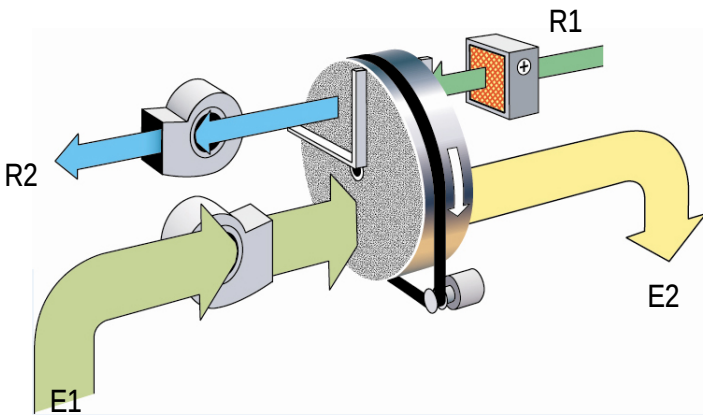
- $t_c < t_{dp}$

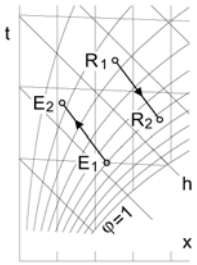
Adsorption dehumidifying

- rotary heat recovery exchangers
- rotor with hygroscopic material as silica gel or activated alumina
- the sorbent material contains a vast number of microscopic pores where water is absorbed
- moisture is condensed and held on the surface of the material
- the adsorbent material can be reactivated by heat - regeneration

Dehumidifying

Sorption unit



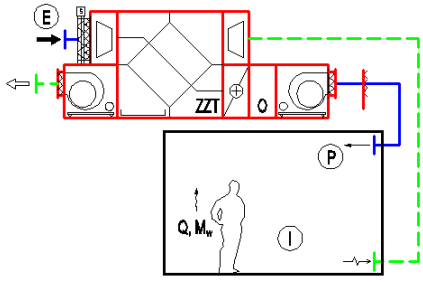


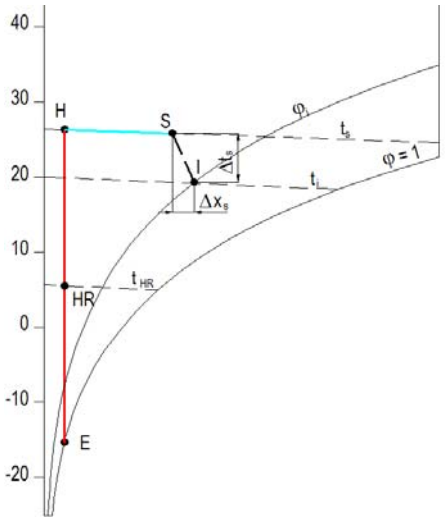
43

Example of winter process

Single duct air system

- heat recovery
- heater
- humidifying (steam)
- conditioning in the room



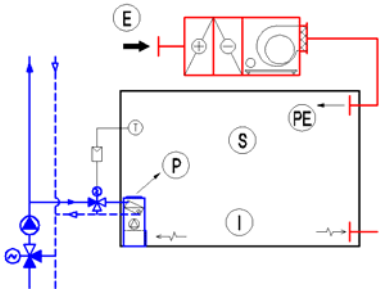


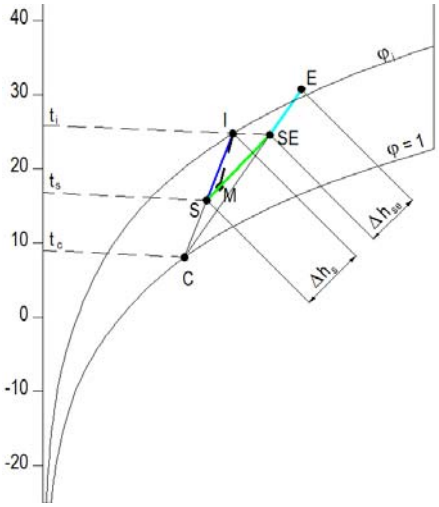
44

Example of summer process

Fan coil units

- outdoor air cooling
- recirculation of air
- mixing in the room
- conditioning in the room





45

FAKULTA STROJNÍ ČVUT V PRAZE

ÚSTAV TECHNIKY PROSTŘEDÍ

Thank you for your attention

utp.fs.cvut.cz